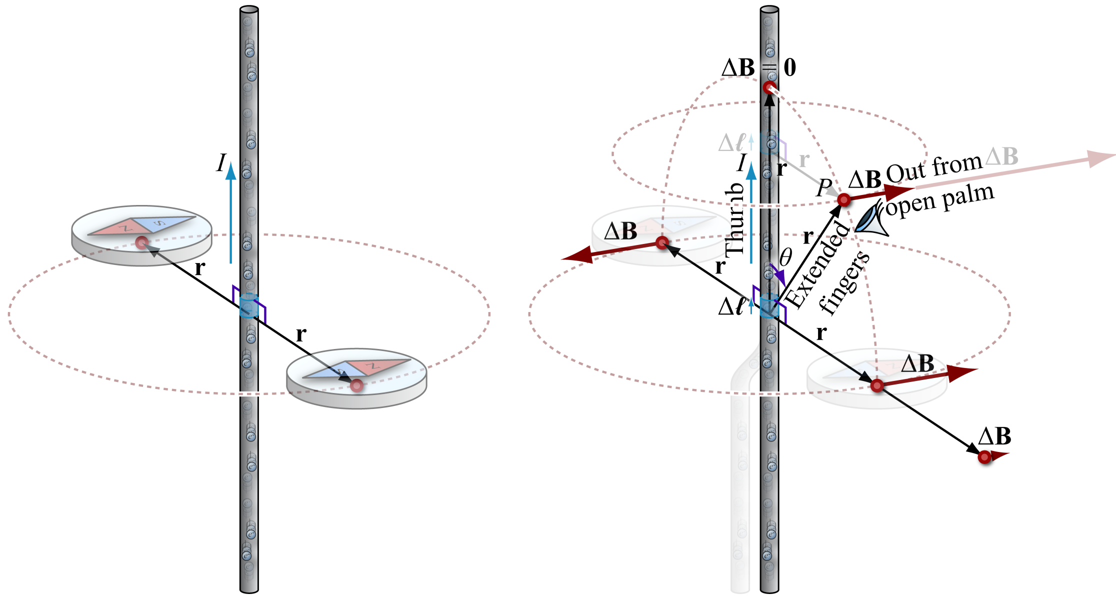
Magnetism (algebra-based physics)

Moving charges create magnetic fields, and magnetic fields exert magnetic forces on moving charges.

Motivating differential form of **Biot-Savart law**



$$\Delta\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{\boldsymbol{\ell}}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\left|\Delta\vec{\boldsymbol{\ell}}\right|\sin\theta}{r^2}\vec{\mathbf{u}}_{\Delta\vec{\boldsymbol{\ell}}\times\hat{\mathbf{r}}}$$

$[\mathrm{B}] = \mathrm{T}$ $\qquad \mu_0 = 4\pi\times10^{-7}\,\frac{\mathrm{T\cdot m}}{\mathrm{A}}$

Superposition $\qquad \vec{\mathbf{B}} = \sum \Delta\vec{\mathbf{B}}$

(All panels illustrate **right** hand rules)

⊙ out (arrowhead)

⊗ in (fletching)

Steps for applying **Biot-Savart law**

Neatly and graphically represent situation(s)

1. Draw current distribution.
2. Draw observation point P.

Graphically represent quantities and their relationships

3. Draw macaroni $\Delta\vec{\boldsymbol{\ell}}$.
4. Draw $\vec{\mathbf{r}}$ from macaroni to P.
5. Draw angle θ between macaroni $\Delta\vec{\boldsymbol{\ell}}$ and $\vec{\mathbf{r}}$.
6. Use RHR to find direction of cross product. Draw $\Delta\vec{\mathbf{B}}$ in this direction.

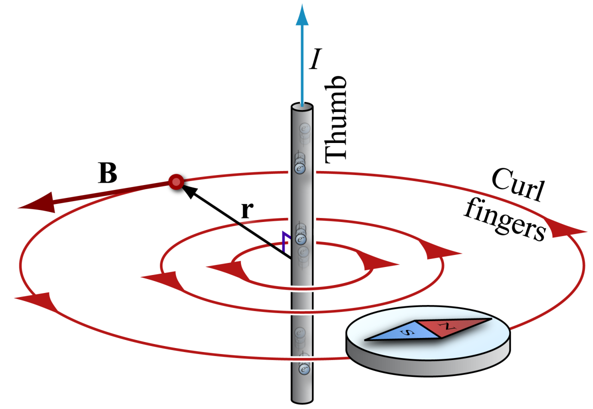
Identify relevant allowed starting point (in)equation(s)

Use numbered steps to show REASoNing

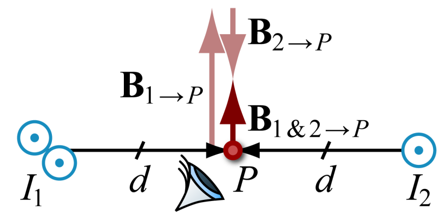
Communicate

Results

Steady infinite line current



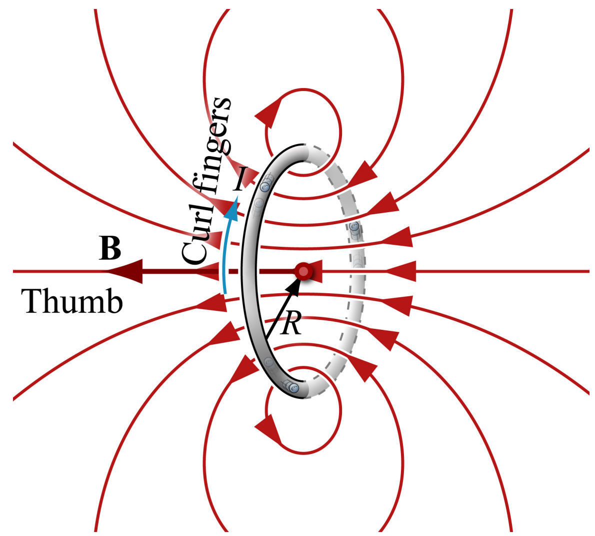
$$\left|\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{2\pi r}$$



Superposition

$$\vec{\mathbf{B}}_{1\&2\&\cdots\to P} = \vec{\mathbf{B}}_{1\to P} + \vec{\mathbf{B}}_{2\to P} + \cdots$$

Steady circular loop current

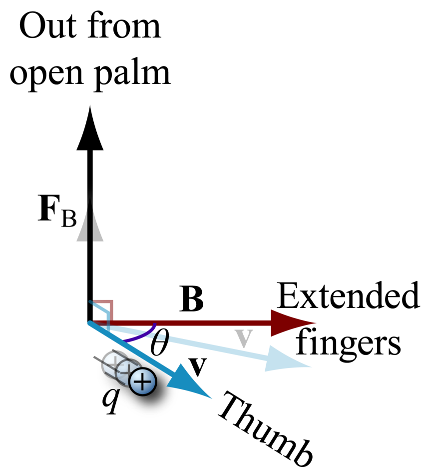


$$\left|\vec{\mathbf{B}}_{\mathrm{CENTER}}\right| = \frac{\mu_0 I}{2R}$$

Magnetism (algebra-based physics)

Moving charges create magnetic fields, and **magnetic fields exert magnetic forces on moving charges.**

Magnetic force on a **moving charge**



(**RHR** for positive moving charge illustrated; for negative moving charge, reverse direction of thumb)

$$\vec{\mathbf{F}}_\mathrm{B} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

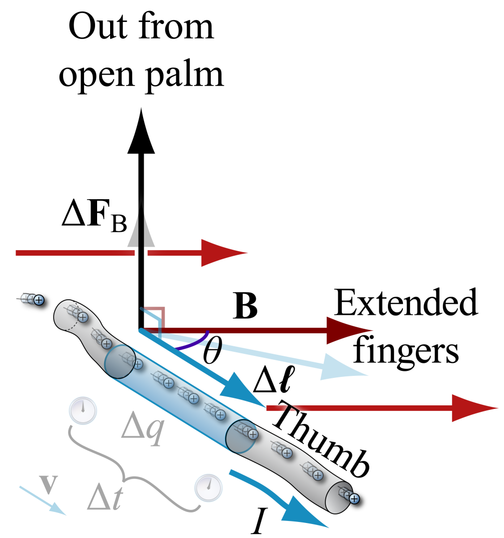
$$\left|\vec{\mathbf{F}}_\mathrm{B}\right| = |q||\vec{\mathbf{v}}| \sin\theta \left|\vec{\mathbf{B}}\right|$$

$$\Delta\vec{\mathbf{F}}_\mathrm{B} = \Delta q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$= \Delta q \frac{\Delta\vec{\boldsymbol{\ell}}}{\Delta t} \times \vec{\mathbf{B}}$$

$$= \frac{\Delta q}{\Delta t} \Delta\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$$

Magnetic force on a **very small current element**



$$\Delta\vec{\mathbf{F}}_\mathrm{B} = I\,\Delta\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$$

$$= I\left|\Delta\vec{\boldsymbol{\ell}}\right|\left|\vec{\mathbf{B}}\right| \sin\theta\, \vec{\mathbf{u}}_{\Delta\vec{\boldsymbol{\ell}}\times\vec{\mathbf{B}}}$$

Steps for applying law for **magnetic force** on a **very small current element**

Neatly and graphically represent situation(s)

1. Draw current distribution.

Graphically represent quantities and their relationships

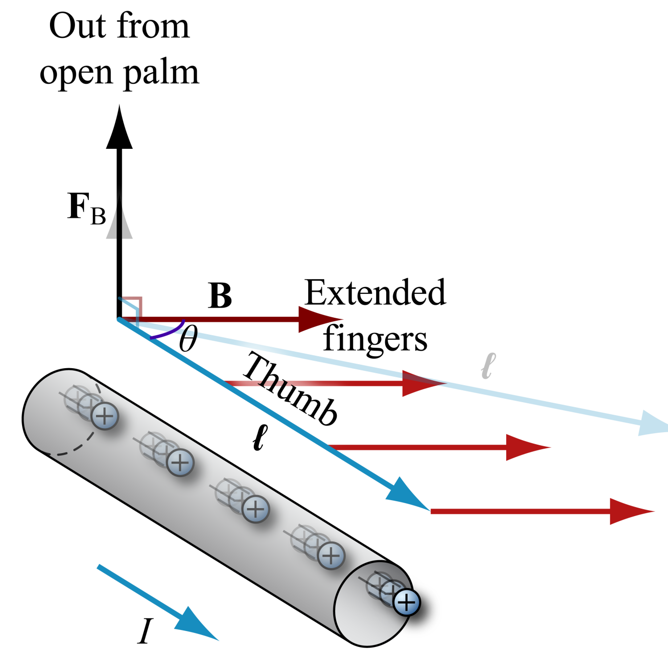
2. Draw background $\vec{\mathbf{B}}$-field.
3. Draw macaroni $\Delta\vec{\boldsymbol{\ell}}$, local $\vec{\mathbf{B}}$, and angle θ between $\Delta\vec{\boldsymbol{\ell}}$ and local $\vec{\mathbf{B}}$.
4. Use RHR to find direction of magnetic force on current element. Draw $\Delta\vec{\mathbf{F}}_\mathrm{B}$ in this direction.

Identify relevant allowed starting point (in)equation(s)

Use numbered steps to show REASoNing

Communicate

Magnetic force on a **line segment of current**



$$\vec{\mathbf{F}}_\mathrm{B} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$$

$$\left|\vec{\mathbf{F}}_\mathrm{B}\right| = I\left|\vec{\boldsymbol{\ell}}\right| \sin\theta \left|\vec{\mathbf{B}}\right|$$